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AN ELECTROMAGNETIC-ANALOGY METHOD OF SOLVING

LIFTING-SURFACE-THEORY PROBLEMS

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AN ELECTROMAGNETIC-ANALOGY METHOD OF SOLVING

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SUMMARY

A method is suggested for making lifting-surface calculations by means of magnetic measurements of an electromagnetic-analogy model. The method is based on the perfect analogy between the strength of the magnetic field around a conductor and the strength of the induced-velocity field around a vortex. Electric conductors are arranged to represent the vortex sheet. The magnetic-field strength is determined by measuring, with an electronic voltmeter, the voltage induced in a small search coil by the alternating current in the wires representing the vortex sheet.

Solutions of nonlinear lifting-surface problems may be obtained by placing the conductors representing the trailing vortices along the fluid lines (Helmholtz condition). A potential-flow solution for the distortion and rolling up of the trailing-vortex sheet may be obtained. By use of the Prandtl-Glauert rule, the lifting-surface theory may be adapted to include first-order compressibility effects.

A comparison was made of the downwash determined by means of a preliminary electromagnetic-analogy model with the downwash obtained by calculation for an elliptic wing having an aspect ratio of 3. The accuracy of the magnetic measurements compared satisfactorily with the accuracy of the downwash calculations.

INTRODUCTION

There are many important aerodynamic problems for which solutions by lifting-line theory are inadequate. These problems can be solved much more satisfactorily by

a lifting-surface theory; that is, a theory in which the
lift is assumed to be distributed over a surface instead
of along a line. The calculations necessary to determine
solutions by lifting-surface theory, however, are rather
laborious even for the simplified case in which the vari-
ation in incremental pressures with the effective camber or
the angle of attack of the surface is linear. A more exact
nonlinear solution is very nearly impossible to calculate
except for a few special cases. A few of the aerodynamic
problems for which solutions by lifting-surface theory are
desired are: the plan-form corrections necessary for the
prediction of finite-span hinge-moment characteristics
from section data; the determination of spanwise and chord-
wise load distributions of wings with low aspect ratio,
wings with sweep, wings in sideslip, wings in roll, and
wings in turning flight; more exact solutions for the
unsteady lift of finite wings; and an improved theory of
the field of flow near propellers.

        In reference 1 it was shown that the plan-form correc-
tions determined from lifting-line theory are inadequate
for hinge-moment predictions. The plan-form corrections
determined by a linear lifting-surface theory (reference 2),
however, were shown to be quite satisfactory for the pre-
diction of hinge moments at small angles of attack. For
wings at larger angles of attack, especially for wings with
square tips, a nonlinear lifting-surface theory is required.

        The electromagnetic-analogy method was developed in
an attempt to make calculations by both linear and non-
linear lifting-surface theories practical. The time and
expense required to build and test an electromagnetic-
analogy model of a wing and wake were expected to be small
compared with the cost of applying other methods available
at present, even for the linear case. The electromagnetic-
analogy method is based on the fact that the magnetic
field around a wire carrying electric current is perfectly
analogous to the velocity field around a vortex. It has
also been shown (reference 3) that the lifting surface and
wake may be represented by a vortex sheet and may therefore
be replaced by conductors arranged in the configuration
of the equivalent vortex sheet. Simple measurements of
the magnetic-field strength then replace the difficult
induced-velocity calculations.

        For nonlinear solutions of lifting-surface problems,
the trailing-vortex sheet represented by the wires is
rolled up and distorted instead of lying in a plane as it

is usually assumed to do. In figure 1 is shown a simplified picture of a rectangular wing of low aspect ratio at a large angle of attack with a rolled-up and distorted trailing-vortex sheet. Of the various features of the distorted vortex sheet that contribute to the nonlinearity, the most important is the vertical spacing of the trailing vortices. The increase in vertical spacing as the angle of attack is increased results in a decrease in the vertical component of induced velocity at the surface, especially near the wing tips; thus the slope of the lift curve is increased as the angle of attack increases (see reference 4) and the slopes of hinge-moment curves are more negative (reference 1).

The present report describes the basic theory of the electromagnetic-analogy method and the general procedure by which various aerodynamic problems may be solved by this analogy. A few preliminary results for the linear case are presented for an elliptic wing having an aspect ratio of 3, as well as a comparison of the results obtained by the present method and the results calculated by the method of reference 2.

SYMBOLS

Γ vortex strength

Γ_{max} maximum vortex strength

Δp pressure difference across lifting surface

V free-stream velocity

M mach number, ratio of free-stream velocity to sonic velocity

ρ fluid density

x distance along free-stream direction from leading edge of wing

y spanwise distance

z vertical distance above plane of vortex sheet

H magnetic-field strength

i   current in conductor

e   induced voltage

l length of conductor or vortex

r   distance from element of conductor or vortex to point
           in question

v   induced velocity

w   vertical component of induced velocity (downwash)

u   horizontal component of induced velocity (free-stream
           direction)

K   constant

t   time

b   span

c   chord

      Bar above symbol indicates a vector, as $\bar{H}$, $\bar{l}$, $\bar{r}$, and $\bar{v}$.

BASIC THEORY

Solution of Aerodynamic Problems by Available

Lifting-Surface Theories

      The distribution of lift over a lifting surface cannot, in general, be expressed in any simple mathematical form such as can be obtained by lifting-line theory. This statement is especially true for nonlinear lifting-surface problems. Except for a few special plan forms (references 5 and 6), the method of determining the induced downwash for a given lift distribution also is too complex for expression in mathematical form. In order to obtain an exact, complete analytical solution, however, such expressions must be known.

      The determination of the surface to sustain an arbitrary lift distribution may be accomplished by means of the electromagnetic-analogy method described herein or, for the linear case, by the semigraphical method of reference 2. The inverse problem, determining the lift distri-

bution over an arbitrary surface, may then be solved by a
process of successive approximations. A reasonable distri-
bution of vorticity is assumed or calculated from the
simple lifting-line and thin-airfoil theories, and the
induced velocities corresponding to that vortex distribution
are determined by making an electromagnetic-analogy model
of the vortex sheet and measuring the magnetic-field strength.
If the induced velocities do not satisfy the boundary con-
ditions - that is, the shape of the lifting surface - the
vortex sheet is suitably altered and the process repeated
until the boundary conditions are satisfied. For the non-
linear problem, not only must the induced velocities satisfy
the boundary conditions of the wing shape but also the
trailing vortices must satisfy the Helmholtz condition,
namely, that the vortices must trail along fluid lines.
(See reference 7.) In practice, satisfying these simple
conditions may require a considerable amount of work unless
the first approximation is fairly accurate. In order to
obtain a somewhat more general solution, for the linear
case at least, the surfaces required to support several
different lift distributions may be determined so that the
shape for a particular lift distribution may be estimated
by a process of interpolation or superposition.

There are, however, several problems for which a com-
plete potential-flow solution of the inverse problem is not
necessary. For example, in order to include the main
effects of viscosity, the estimation of the hinge-moment
parameter for finite-span wings should be made by applying
theoretical aspect-ratio corrections to experimental section
hinge-moment parameters. For such problems the additional
aspect-ratio corrections may be determined simply and
accurately from the surface required to support a given
lift distribution (reference 1) as found by lifting-surface
theory.

The results of the electromagnetic-analogy solution
of the lifting-surface theory may be corrected for first-
order compressibility effects by a simple application of
the Prandtl-Glauert rule (reference 8). The method consists
of determining the incompressible-flow characteristics of
an equivalent wing the chords of which are increased by the
factor $\frac{1}{\sqrt{1 - M^2}}$. It is therefore necessary only to build
an electromagnetic-analogy model of a wing of this slightly
lower aspect ratio $\left(\text{lower by the factor } \frac{1}{\sqrt{1 - M^2}}\right)$ or to
test models of several aspect ratios and interpolate. The

pressures (or vorticity) acting upon this incompressible
equivalent of lower aspect ratio, however, must be increased
by the factor $\dfrac{1}{\sqrt{1 - M^2}}$. In order to find the lift, these
increased pressures are referred to the original wing and
integrated.

Vortex Sheet

Inasmuch as the equivalence of a lifting wing and wake
to a vortex sheet may be considered to be well established
(reference 3), only the important characteristics of the
equivalent vortex sheet and the relations between the lifting
wing and the vortex sheet will be given.

The part of the vortex sheet representing the lifting
wing consists of a sheet of bound vortices. The strength
of the vortices is directly associated with the lift distri-
bution of the wing. The product of the air density, the
free-stream velocity, the vortex length perpendicular to
the free-stream velocity, and the vortex strength of each
elementary vortex equals the lift contribution by that
elementary vortex (Kutta-Joukowski law). If the lift
distribution of the wing is known or assumed, therefore,
the equivalent vortex distribution may be easily obtained.
A continuous lift distribution (as measured by pressure
distribution Δp) may be integrated to give a continuous
vortex distribution. The integration formula (reference 2)
for obtaining the vortex distribution is

$$\Gamma = \int_0^x \frac{\Delta p}{\rho V}\, dx \qquad\qquad (1)$$

where ρV is the product of the density and the free-
stream velocity and the integration is made in the free-
stream direction. Equation (1) gives the chordwise
Γ-function at each section. The values of Γ at the
trailing edge of the wing at each section also give the
spanwise vortex distribution of the wake. The bound
vortices may be assumed to lie along a mean surface, half-
way between the upper and lower surfaces of the wing.

The part of the vortex sheet representing the wake
consists of the so-called trailing vortices. As the name
implies, these vortices originate at the trailing edge of
the wing and merely trail behind the wing. These vortices
are free to move and thus lie along the local stream lines,
or fluid lines. This simple kinematic condition, the
Helmholtz condition (reference 7), determines the configu-
ration of the trailing-vortex sheet.

The trailing vortices for lightly loaded wings usually lie very near a plane; that is, these vortices travel almost straight back from their origin at the trailing edge of the wing. For highly loaded wings, however, the trailing-vortex sheet is known to be considerably distorted, rolled up, and inclined with respect to the free-stream direction. (See fig. 1.) The characteristics of the air flow behind wings are described in more detail in references 9 and 10.

Electromagnetic Analogy

The perfect analogy that exists between the strength of the magnetic field around conductors and the strength of the induced-velocity field around columnar (finite-diameter) vortices is well known. In fact, the phenomena of the induced velocities around vortices are usually explained in aerodynamic textbooks by the analogy with electromagnetic phenomena. Both phenomena are potential flows.

The vector form of the differential equation for the magnetic-field strength $d\overline{H}$ at any point caused by the current i flowing in an infinitesimal length $d\overline{l}$ of wire is (from p. 242 of reference 11)

$$d\overline{H} = i\frac{d\overline{l} \times \overline{r}}{|\overline{r}|^3} \tag{2}$$

where $\overline{r}$ is the vector from the current element to the point in question. This equation is usually called the Biot-Savart law in aerodynamic textbooks.

The same form of equation (2) but with different constants applies to the induced velocity $d\overline{v}$ at any point caused by an infinitesimal length $d\overline{l}$ of a vortex of strength Γ (reference 9); that is,

$$d\overline{v} = \frac{\Gamma}{4\pi} \frac{d\overline{l} \times \overline{r}}{|\overline{r}|^3} \tag{3}$$

The units in which the various quantities in equations (2) and (3) are usually measured are widely different. In equation (2), for example, H is usually given in gauss, i in abamperes, and l and r in centimeters. In equation (3), v is usually in feet per second, Γ in feet squared per second, and l and r in feet.

Small search coils are used to measure the strength
of the magnetic field. These search coils must be cali-
brated in magnetic fields of known strength - for example,
in a Helmholtz coil (fig. 2). If the vortex equation (3)
and the usual vortex units are used to compute the induced
velocity in the Helmholtz coil (p. 269, reference 11) -
that is, are used as the calibrating unit - and if the
ammeter measuring the current in the Helmholtz coil is
considered to read vortex strength, conversions of electro-
magnetic to aerodynamic units will not be necessary. All
conversions and constants become merely a part of the over-
all calibration constant of the search coils.

CONSTRUCTION OF ELECTROMAGNETIC-ANALOGY MODELS

Approximate Representation of Continuous Vortex Sheet

The replacement of a continuous vortex distribution
by a finite number of conductors must, of course, involve
some approximation. Two procedures for constructing models
have been tried. For the preliminary electromagnetic-
analogy model, a set of 50 circular electric wires carrying
the same current (connected in series) representing
50 columnar vortices of equal strength were distributed
over the wing and wake. (See fig. 3.) The arrangement
of the wires was determined as follows: Fifty contour
lines of Γ were calculated from equation (1). (See
reference 2.) Each wire was placed halfway between two
adjacent Γ-contour lines and thus represented
$\Delta\Gamma/\Gamma_{max} = 0.02$. In order to illustrate the degree of
the approximation involved, the continuous chordwise
Γ-function at the plane of symmetry and the stepwise
distribution of wires used to represent the continuous
distribution are given in figure 4.

The possibility of errors resulting from the use of
an incremental distribution is, however, more serious than
simply the possibility of not obtaining a good represen-
tation of the distribution of vorticity in the continuous
wake. The induced velocity resulting from an incremental
vortex pattern may vary greatly about the mean value that
would be obtained by the continuous sheet, because the
magnitude of the induced velocity varies inversely with
distance from the vortex core and becomes higher than the
mean value on one side of each stepwise vortex increment
and lower than the mean value on the other side. The

effect is illustrated in figure 5. It is thus necessary
to use as many wires as practicable to approach as nearly
as possible a continuous sheet and to measure the induced
velocities at a great number of points so that the mean
value of the induced velocity may be obtained more easily
and more accurately from faired curves.

        The preliminary model, constructed of 50 wires, gave
satisfactory results and it is believed that 50 is about
the optimum number of wires. Construction difficulties
are too great if more wires are used, and the accuracy is
not great enough if fewer wires are used. Another, more
complex method of construction was adopted for a few
models but the results obtained were not much more accurate
than those obtained with the simpler wire construction
method. The other construction method was to use thin
aluminum strips (resulting in "flat wires") rather than
circular copper wires (fig. 6), the reason for this type
of construction being that a more nearly continuous distri-
bution of vorticity and a corresponding smoother induced-
velocity field could be obtained. The effect of eddy
currents in the aluminum strips proved to be more important
than was originally expected, however, and the induced
magnetic field was only slightly smoother than with
circular wires and was not so regular; thus difficulties
in fairing the measured induced magnetic field proved to
be about the same for both types of model.

            Correction for Finite Thickness of Wires

        Calculations by the lifting-surface theory are usually
made for points in the plane of the vortex sheet. Because
the wires representing the vortices are of finite thickness,
the magnetic-field strength must be measured at several
vertical heights and extrapolated to zero, that is, to the
center of the wires. Except near the wing tips and the
leading edge of the wing, this extrapolation is usually
linear and can thus be made quite accurately. The fact
that measurements are made above the wires simplifies the
fairing problem, because the variation in the magnitude
of the vertical component of induced velocity about the
mean value (fig. 5) decreases with increase in vertical
height.

Correction for Finite Length of Trailing-Vortex Sheet

For the steady-state condition of a finite-span lifting surface, the trailing-vortex sheet extends from the trailing edge of the wing infinitely far downstream. It is necessary to determine corrections for the finite length of trailing-vortex sheet of the electromagnetic-analogy models. The wires representing the incremental vortices were connected in series; the closing loops were about 0.7 span behind the wing for one model and about 2 spans behind the wing for another model. (See fig. 3.) Because the correction for the approximation of the infinite length of the trailing-vortex sheet is fairly small - about a 5-percent correction for a 0.7-span wake and less than 1-percent correction for a 1.5- to 2.0-span wake - it appears that wakes need not be longer than 1.5 spans. More downwash is contributed by the closing loops than by the missing trailing vortices. The correction is simply the difference between the down-wash contribution of the closing loops and the contribution of the missing part of the trailing-vortex sheet. The corrections are small and can usually be estimated by assuming that the span loading is a simple rectangular loading or the sum of two rectangular loadings.

The corrections may also be determined from measurements of the induced field at a distance behind the closing wires equal to the length of wake represented. That is, if a measurement is made at the corresponding spanwise point 1 wake length behind the model, the effect will be the same as if a measurement were made on the wing of the downwash due to a mirror image of the model reflected from the closing loops. Such an image would cancel the effect of the closing loops and would double the length of the wake. The remaining error is relatively small.

SUGGESTED METHOD OF MEASURING MAGNETIC-FIELD STRENGTH

Several methods of measuring the magnetic-field strength were investigated. The fundamental principle of the method selected as being the simplest and as requiring the least special equipment and the least development work is given herein. This method consists in passing alternating currents through the wires representing the vortex sheet and measuring the magnetic-field strength by means of the

voltage induced in a small search coil. A discussion of
other possible methods of measuring the magnetic-field
strength is given in the appendix.

Basic Principles

According to the principles of electromagnetic
induction (reference 11), the electromotive force induced
in a fixed circuit (the search coil, in this case) by
changing the magnetic flux through the coil is equal to
the time rate of change of flux linkages with the coil.
If an alternating current is passed through the wires
representing the lifting wing and the wake, the magnetic-
field strength will also be alternating at the same
frequency and with the same wave shape. This fact may be
seen from equation (2), because the instantaneous value
of $d\bar{H}$ is proportional to the instantaneous value of i.

A simple method of determining the magnetic-field
strength, therefore, is to measure with an electronic
voltmeter the voltage induced in a small search coil (fig. 7)
when alternating current is passed through the wires
representing the vortices (fig. 3). The search coil is
directional; that is, this coil measures only the component
of magnetic-field intensity along the axis of the coil.

Practical Problems

Upper-frequency limit.- Because the voltage induced
in the search coil is proportional to the rate of change
of flux, it would seem desirable to use a very high-
frequency current so that the voltage induced in the
search coil would be very large and thus somewhat easier
to measure. The maximum frequency that may be used,
however, is about 300 cycles, because the capacitance
between the closely spaced wires in the model representing
the vortex sheet becomes important above this frequency.
If greater frequencies are used, capacitative reactance
becomes sufficiently low to have perceptible effect. When
the capacitative reactance between wires becomes small,
the current is not the same in all wires even though they
are connected in series. The maximum allowable frequency
was determined from measurements of the capacitative
reactance of the electromagnetic-analogy model used for
the preliminary tests, to be described in the section
"Preliminary Tests with Electromagnetic-Analogy Model."
The capacitative reactance, and thus the leakage current
of this model, became measurable above about 300 cycles.

Several other models tested have had about the same or a
higher limiting frequency; therefore, a 300-cycle limit
is believed to be conservative.

Wave shape.- The fact that the frequency is limited
to about 300 cycles also requires that the wave shape of
the current in the vortex sheet be very nearly sinusoidal;
that is, any departure from a simple sine wave means that
higher harmonic frequencies are also present. Usually the
most nearly sinusoidal wave shape is obtained by putting
enough condensers in series with the model to balance the
inductance of the model; in this way a pure resistance
load is put on the generator.

The easiest way to determine the wave shape is to look
at the voltage output of the search coil by means of an
oscilloscope. This wave shape is not that of the current
in the wires but depends on the rate of change of current i
with time t - that is, the induced voltage e is equal to
$K \frac{di}{dt}$. The amplitude of the third harmonic as seen on the
oscilloscope is, then, three times the amplitude of the
actual third harmonic of the current. In other words, the
wave shape of the voltage output of the search coil looks
much more irregular than the wave shape of the current in
the wires actually is. If the filters that are added to
the circuit make the voltage output of the search coil
appear satisfactory, no further test is necessary.

Practically all alternators have wave shapes that are
not sinusoidal and that change with load. The wave form
of the current in the Helmholtz coil, used to calibrate
the search coil, must be identical with that of the current
in the electromagnetic-analogy model. Probably the easiest
way to make these wave forms identical is to connect the
Helmholtz coil and the electromagnetic-analogy model in
series. The Helmholtz coil and the electromagnetic-
analogy model should be placed as far apart as possible
and should be oriented in such a way that downwash measure-
ments on the analogy model are unaffected by the field of
the Helmholtz coil and that the Helmholtz field is unaltered
by the analogy-model field while the search coil is being
calibrated. It would also be advantageous to effect an
arrangement such that the search-coil calibration could
be checked frequently. A fairly satisfactory arrangement
was employed for the preliminary tests. (See fig. 8.)
The leads from the search coil were long enough to go to
either the Helmholtz coil or the analogy model.

Search coils.- The optimum size of the search coil
depends upon two factors. First, the search coil must be
small enough to make "point" measurements possible. If
the coil is too large and the magnetic-field strength varies
nonlinearly with position, the effective center of the
coil may be too far from the geometric center. If the
coil dimensions are kept small relative to the wing, little
error due to this cause will result. The size of the
search coil therefore depends upon the size of the
electromagnetic-analogy model. The second consideration
is that, for accurate voltage measurements, the voltage
output of the search coil should be at least 0.0001 volt
and a minimum value of 0.001 volt is preferable.

Several sizes of coil were tried with the preliminary
model, and it is felt that the maximum-size search coil
that gives measurements fairly close to point measurements
is one with a diameter about 1 percent of the wing semi-
span and a height about 0.5 percent of the wing semispan.
In order to get the desired search-coil voltage output,
the model semispan must be at least 3 to 4 feet and the
search coil should have about 1000 turns and be wound with
about No. 43 wire (0.0022-in. diam.). If a larger model
is used, larger wire may be used to wind the coil, although
wire as small as 0.001 inch was wound satisfactorily for
the coil used for the preliminary tests. It is probably
desirable to have a number of search coils (fig. 7) of
various sizes, however, to meet any special conditions.
Smaller coils are desirable near the leading edge and the
tips of the wing and in other places where the flux field
varies rapidly.

The search coil must be wound rather carefully so
that all loops are as nearly perpendicular as possible to
the axis of the coil in order to maintain the directional
properties of the coil. The coil must then be carefully
mounted on the survey apparatus. A test for the correct
mounting (or alinement) of the coil is to read the voltage
output of the small coil when mounted with its axis vertical
at a position on the electromagnetic-analogy model where
the horizontal component of the magnetic field is stronger
than the vertical component. Regardless of how the search
coil is turned about its vertical axis, the voltage out-
put should be the same if the search coil is kept at the
same place on the model.

The leads from the search coil to the electronic
voltmeter must be twisted so that no large loops are present.

Otherwise, the voltage measured by the electronic volt-
meter will not only be the voltage induced in the search
coil but will also consist of the voltage induced in these
leads. The errors resulting from pickup in the leads may
be made negligible by tightly twisting the leads, by
making the number of turns in the search coil large so
that the lead pickup gives a small percentage error, and
by bringing the leads in perpendicular to the vortex sheet
so that the leads will be in the region of low magnetic-
field strength.

Extraneous fields.- One of the most important problems
in measuring the magnetic-field strength is to filter out
all extraneous fields. It is desirable to make the tests
in a wooden building fairly far from electrical distur-
bances such as electric motors and computing machines.
Reasonably satisfactory results may be obtained in spite
of these disturbances, if necessary, by the use of an
electrical filter in the circuit of the search coil and
electronic voltmeter. Such a filter suppresses any voltage
of frequencies other than the one passing through the
vortex sheet. Because the filter may act as a search coil
itself, it must be located some distance from the model.

Measuring equipment.- No current should be drawn in
measuring search-coil voltage, which is of the order of
only 1 millivolt. Commercial electronic voltmeters combine
the high sensitivity and high resistance demanded. Small
amounts of power-supply ripple usually exist in these
voltmeters. This power-supply ripple interferes with
measurements made at integral multiples of the line fre-
quency, and care should be taken to avoid use of these
frequencies in testing.

PRELIMINARY TESTS WITH ELECTROMAGNETIC-ANALOGY MODEL

In order to check the accuracy of the electromagnetic-
analogy method of solving lifting-surface problems, a
vortex pattern for which the induced velocities had already
been calculated by the method of reference 2 was inves-
tigated. Calculations were made of the vertical component
of the velocities induced by a plane vortex sheet repre-
senting the lift distribution estimated from the two-
dimensional theories for an elliptic wing having an aspect
ratio of 3.

Equipment

A small model of the plane vortex sheet was constructed of 50 wires representing 50 incremental vortices. This model (fig. 3) represents about the smallest model (wing span of 3.3 ft) that will yield satisfactory results. A small search coil (fig. 7(a)) of 1000 turns was used to measure the magnetic-field strength. A Helmholtz coil (fig. 2) was used to calibrate the search coil. The power supply was a 500-cycle alternator driven by a direct-current motor, the speed of which was controlled so that the frequency output was held at 270 cycles. The direct-current field of the alternator was adjusted to give a current output of 8 amperes. The wave shape was very nearly sinusoidal (third harmonic, less than 4 percent of the first harmonic).

Some trial tests were made in the workshop of the Langley Atmospheric Wind Tunnel Section. The results proved unsatisfactory, however, because of excessive electric and magnetic interference. The apparatus was then moved to a large wooden building, in which there was very little electric and magnetic interference. The setup is shown in figure 8. The small amount of 60-cycle electric interference that was still present was eliminated by using a 100-cycle high-pass filter in the electronic-voltmeter circuit.

Results

A complete survey was made of the induced-velocity field (both the horizontal and vertical components) at from 50 to 100 chordwise points at each of 15 spanwise locations on the model and at several vertical heights. Near the leading edge and the tips of the model, the effect of vertical height was very large and, at all locations, the effect was large enough to require surveys at several heights in order that the results could be extrapolated to zero vertical height.

The most important component of induced velocity computed by lifting-surface theory is the vertical component. The only reason for measuring the horizontal component is to check the arrangement of the wires representing the vortices. According to the assumptions of thin-airfoil theory, the horizontal component of induced velocity is proportional to the pressure distribution. Values of the

horizontal component u of the induced velocity
measured at a relative vertical height of $\frac{z}{b/2} = 0.008$
are shown in figure 9 along with the theoretical velocity
distribution that the wing was built to represent. The
agreement is satisfactory and indicates that the model
was constructed with sufficient accuracy.

Chordwise surveys of the measured value of the ver-
tical component w of the induced velocity are presented
in figure 10 for several spanwise locations and vertical
heights. (Not all of the data are presented.) These and
similar data were extrapolated to zero vertical height
(fig. 11) and corrected for the finite length of the
trailing-vortex sheet (correction, about one-half of 1 per-
cent) and are summarized in figure 12. Included in fig-
ure 12 are the calculated values. The agreement between
the two sets of results may be seen to be satisfactory.
The time and labor involved in obtaining the solution were
considerably less (approximately one-third the man-hours)
by the analogy method than by calculation, after the proper
experimental technique had been determined.

CONCLUDING REMARKS

A method for making lifting-surface calculations by
means of magnetic measurements of an electromagnetic-
analogy model has been developed. The method is based on
the perfect analogy between the strength of the magnetic
field around a conductor and the strength of the induced-
velocity field around a vortex. Electric conductors are
arranged to represent the vortex sheet. The magnetic-
field strength is determined by measuring, with an elec-
tronic voltmeter, the voltage induced in a small search
coil by the alternating current in the wires representing
the vortex sheet.

A comparison was made of the downwash determined by
means of a preliminary electromagnetic-analogy model with
the downwash obtained by calculation for an elliptic wing
having an aspect ratio of 3. The accuracy of the magnetic
measurements compared satisfactorily with the accuracy of
the downwash calculations.

Other applications of the method include solutions
of nonlinear lifting-surface problems obtained by placing

the conductors representing the trailing vortices along
the fluid lines (Helmholtz condition). A potential-flow
solution for the distortion and rolling up of the trailing-
vortex sheet may be obtained. By use of the Prandtl-
Glauert rule, the lifting-surface theory may be adapted
to include first-order compressibility effects.

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    National Advisory Committee for Aeronautics
        Langley Field, Va.

APPENDIX

ALTERNATE METHODS OF MEASURING MAGNETIC-FIELD STRENGTH

By R. A. Gardiner

Several methods could be used to make the measurements described in this report. The methods that were considered are:

Pickup Device	Measuring Instrument

1. With d-c. field on wing

    a. Search coil (flip coil or collapse of field)   Ballistic galvanometer

    b. Rotating search coil

        With slip rings   A-c. electronic voltmeter

        With commutator   D-c. amplifier and voltmeter

    c. Saturated-core magnetometer   Suitable electronic equipment

    d. Torsion-type magnetometer   Suitable optical equipment

2. With a-c. field on wing

    a. Saturated-core magnetometer   Suitable electronic equipment

    b. Bismuth bridge   Wheatstone bridge

    c. Search coil   A-c. electronic voltmeter

The considerations that led to the choice of the method used (method 2c in the foregoing list) were the simplicity, the sturdiness and availability of the equipment, the development work required, the probable success and accuracy, the magnitude of field strength required, the minimum possible size of the pickup device, and the freedom from interference of stray fields. The outstanding

advantages or disadvantages of the various methods may be summarized as follows:

Method 1a.- The use of a search coil and ballistic galvanometer is the established method of magnetic-field measurement. In order to secure the necessary sensitivity, however, a rather delicate galvanometer would have to be used and would probably require a special vibration-free support. The flip coil would require tare readings of the earth's magnetic field. With a stationary coil and collapsing field, the inductance of the wing would prevent the desired instantaneous collapse, the galvanometer would not be used in a true ballistic manner, and errors would result.

Method 1b.- A rotating search coil and associated equipment have been used to make magnetic measurements; however, the induced voltages to be measured are lower than those usually measured by this method. The necessary sliding contacts would probably introduce thermal electromotive forces and variable resistance and would be subject to corrosion. Precision machine work would be necessary to minimize difficulties from the motor and bearings.

Methods 1c and 2a.- It is known that a coil containing a high-permeability metal (Permalloy or Mumetal) will have a large induced voltage across its terminals as the core becomes magnetically saturated. This induced voltage is due to the great change in inductance that occurs at the saturation point. This principle has been used in the measurement of magnetic fields; the field to be measured is superimposed upon a field set up in the core and the change in voltage due to saturation is measured. Although this method can be made very sensitive, a large amount of electronic equipment is necessary and the presence of a ferromagnetic substance might cause distortion of the field.

Method 1d.- The use of torsion magnetometer is an accurate method of measuring the earth's field. A small magnet suspended by a suitable fiber is deflected by the earth's field. This deflection is measured and, from the known magnetic moment of this magnet, the earth's field may be determined. The measuring element is very sensitive and the time necessary to take one reading is rather long. In addition, such an instrument would probably be of delicate construction.

Method 2b.- Resistance change due to the presence of magnetic lines takes place in bismuth. Measurement of the resistance change by use of a bridge is a possible method of measuring the magnetic-field strength. At present a powerful magnetic field is necessary in order to make practical measurements. Considerations of the available power supply eliminated this method.

Method 2c.- The method finally selected - that using an a-c. voltmeter, an a-c. field, and a small search coil for the pickup device - appeared to be the means which would be least troublesome and which would use easily available and simple components. Details of this method are given in the report.

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Figure 1. - Illustration of the rolling-up of the trailing-vortex sheet behind a rectangular wing of low aspect ratio.

Electric wires wound around these hoops

Distance between coils
equals radius of coils

Radius

Wooden
framework

Twisted leads
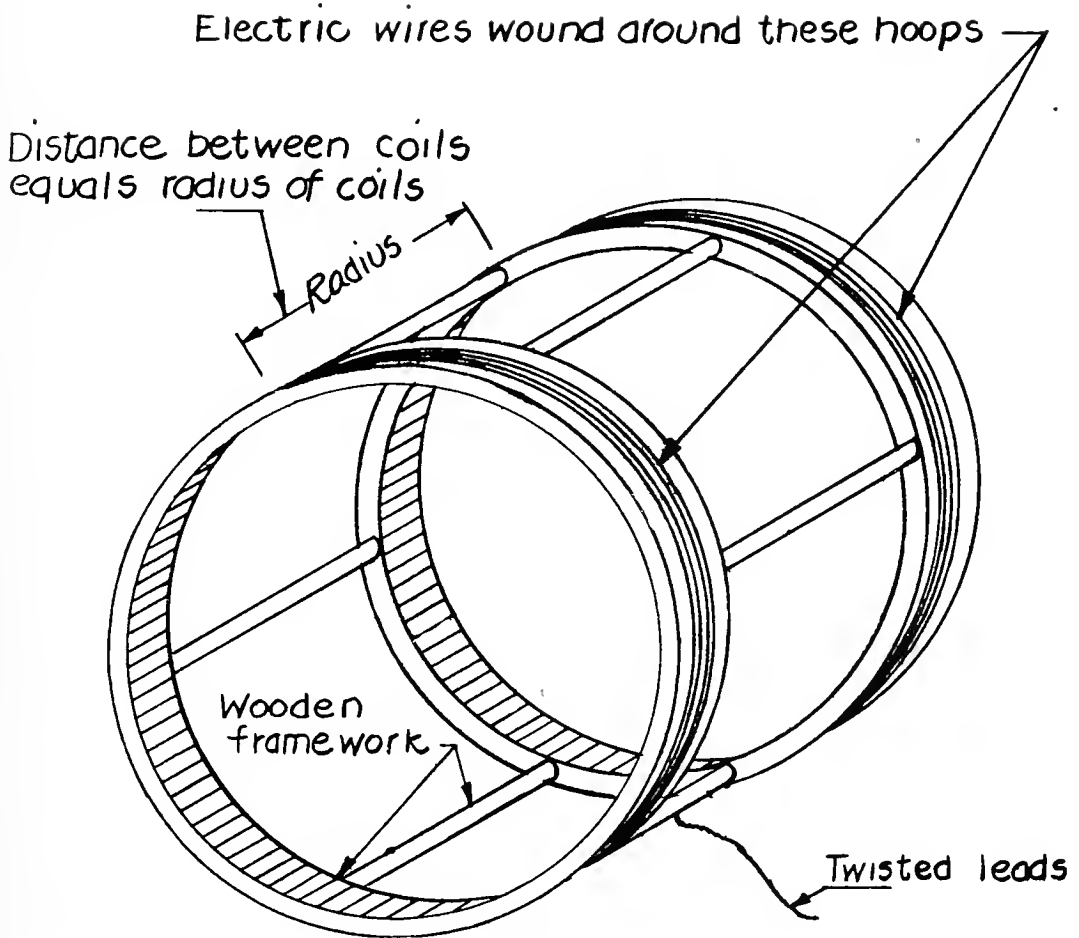
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Figure 2.- A Helmholtz coil. The search
coil to be calibrated is located in
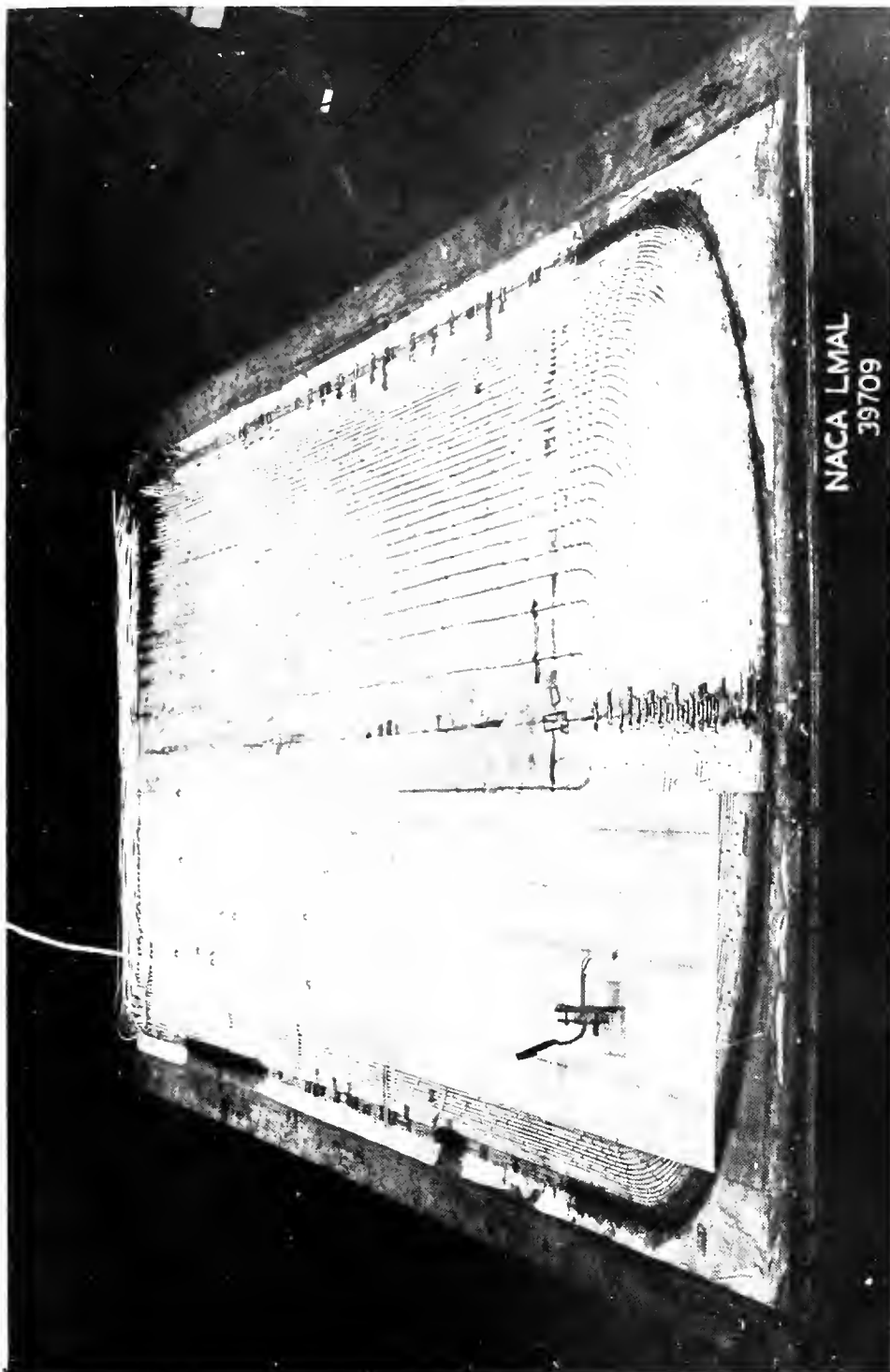the geometric center of the Helmholtz
coil.

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Figure 3.- Preliminary electromagnetic-analogy model of elliptic wing of aspect ratio 3 with flat-plate-type loading. (Wake represented for distance equal to about twice wing span.)
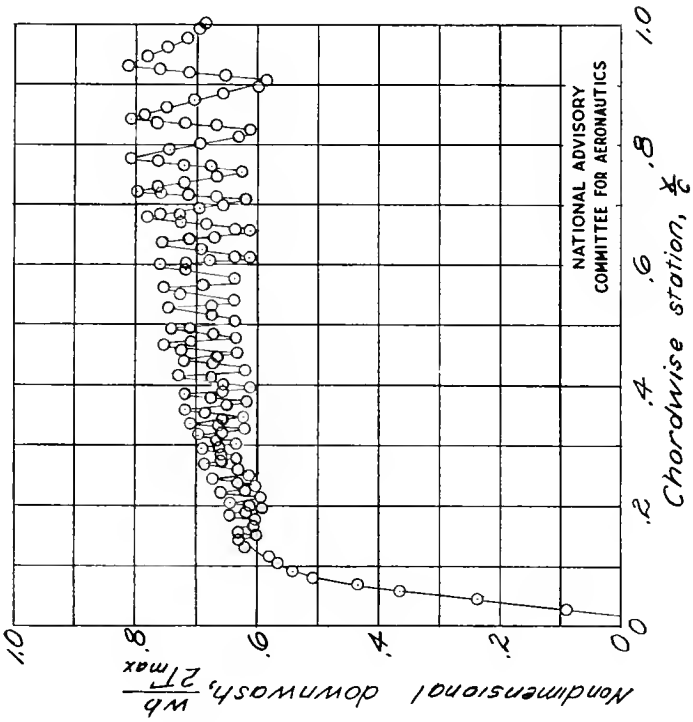
Figure 5.- Chordwise distribution of the nondimensional downwash $\frac{wb}{2\Gamma_{max}}$ at $\frac{y}{b/2} = 0$ and $\frac{z}{b/2} = 0.011$. Flat-plate type of loading.
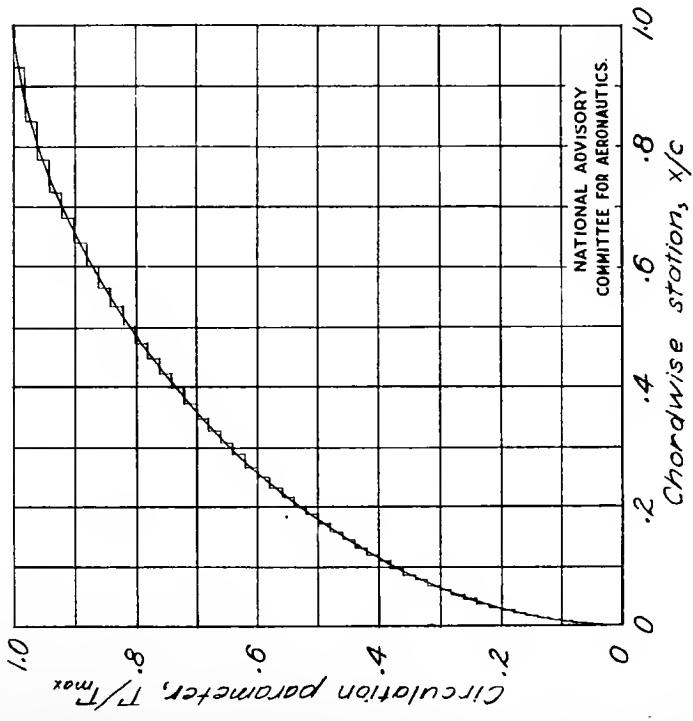
Figure 4.- Continuous and stepwise distribution of the circulation parameter Γ/Γ_{max} at the plane of symmetry; 50 steps.

Figure 6.- An electromagnetic-analogy model constructed of flat aluminum strips. (Model represents one semispan of an elliptic wing having an aspect ratio of 3 with elliptic chord loading.)
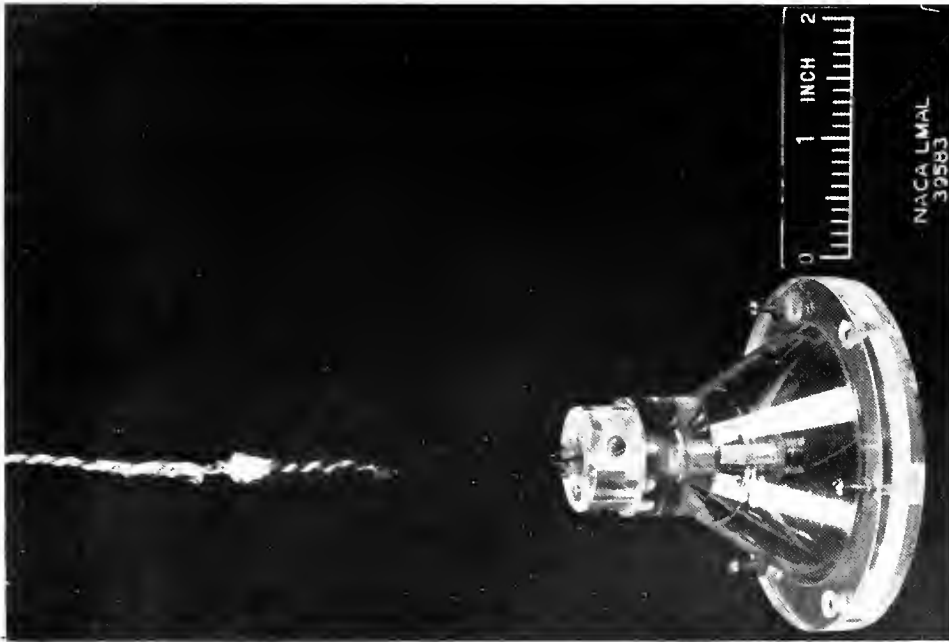
(b)
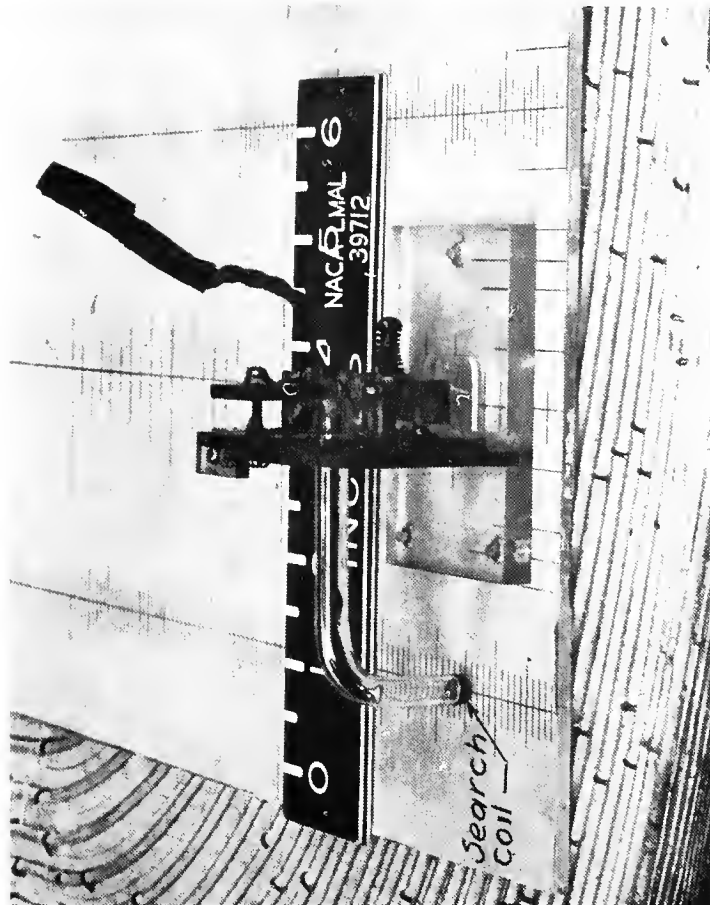
(a)

Figure 7.- Representative search coils.

Figure 8.- General arrangement of electromagnetic-
analogy experiment.
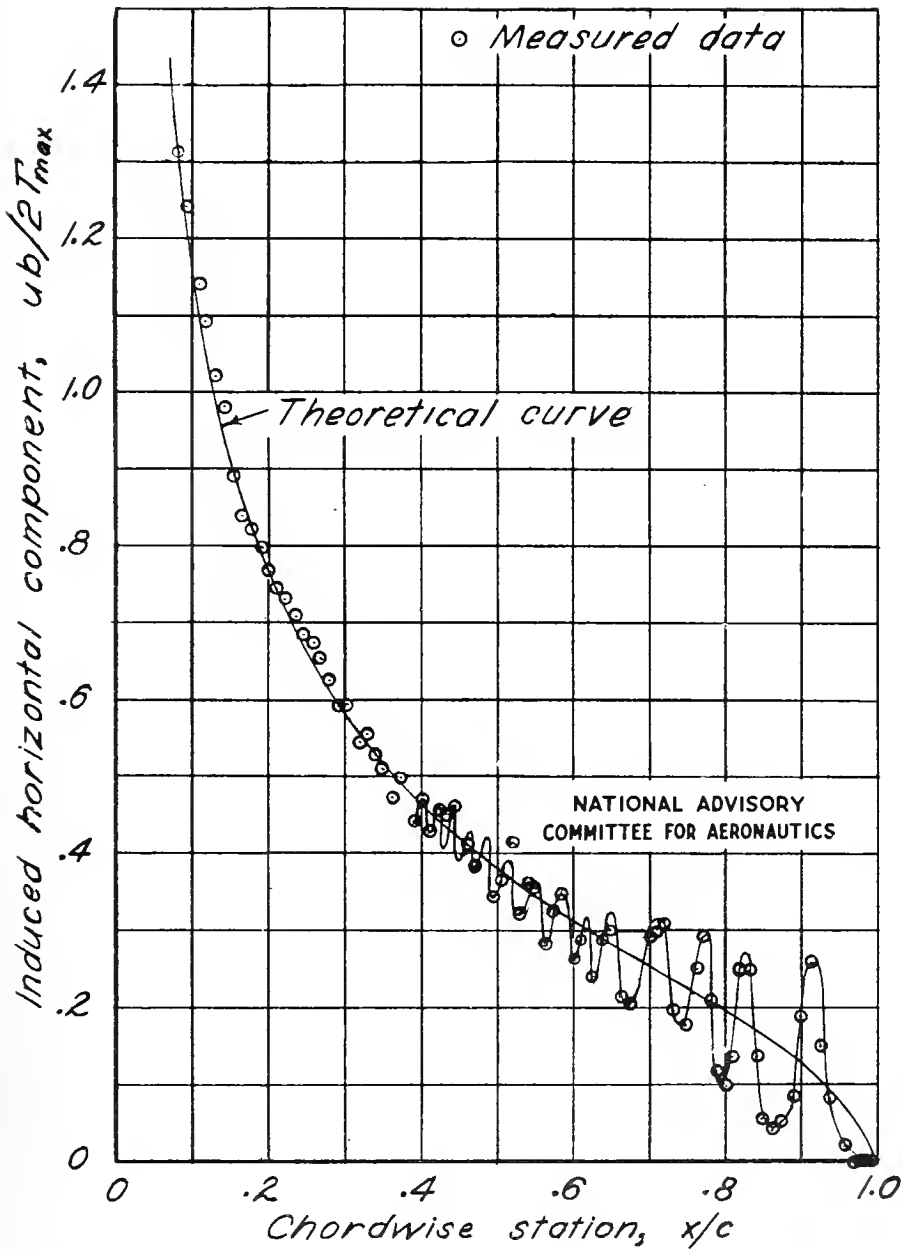
Figure 9.— Chordwise distribution of the nondimensional horizontal velocity component $ub/2\Gamma_{max}$ at $\frac{y}{b/2}=0$ and $\frac{z}{b/2}=0.008$. Flat-plate type of loading.
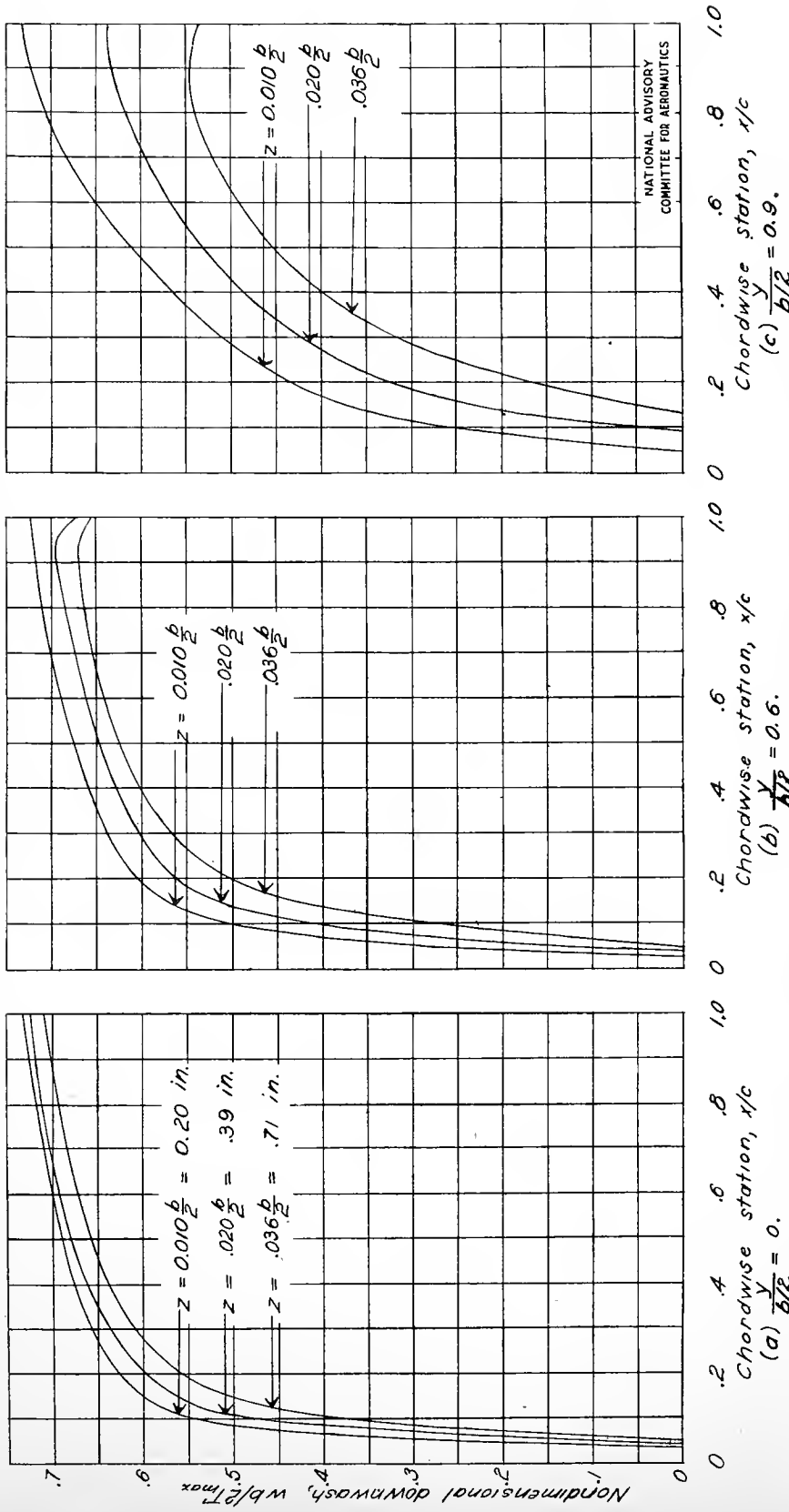
Figure 10.- Induced downwash for several vertical heights at several spanwise stations for an elliptic wing having an aspect ratio of 3 with flat-plate type of loading. Faired curves from measured data.
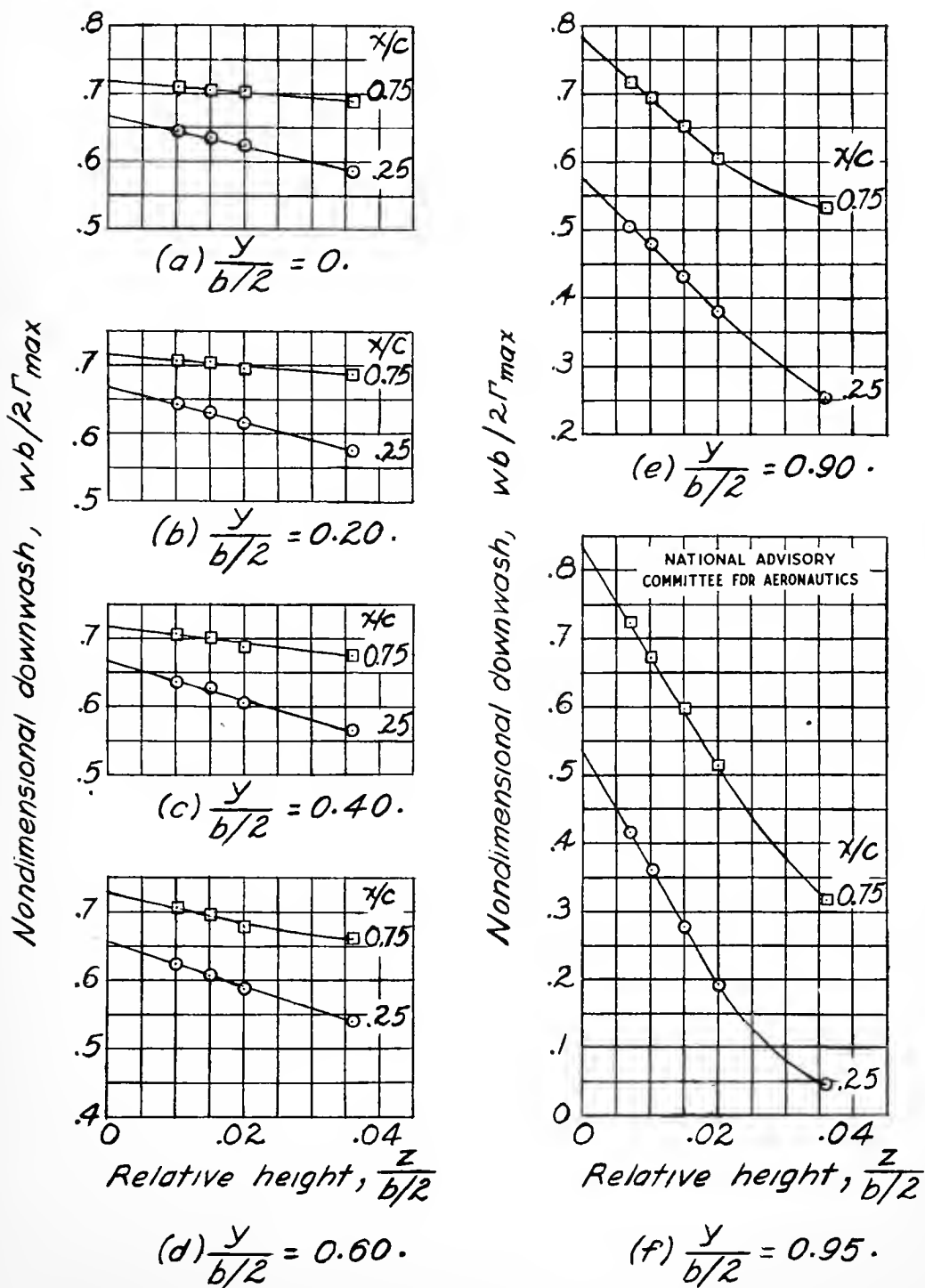
(a) $\frac{y}{b/2} = 0$.

(b) $\frac{y}{b/2} = 0.6$.

(c) $\frac{y}{b/2} = 0.9$.

Figure 11.— Extrapolation to zero vertical height of data obtained from an electromagnetic-analogy model of an elliptic wing having an aspect ratio of 3. Flat-plate type of loading.
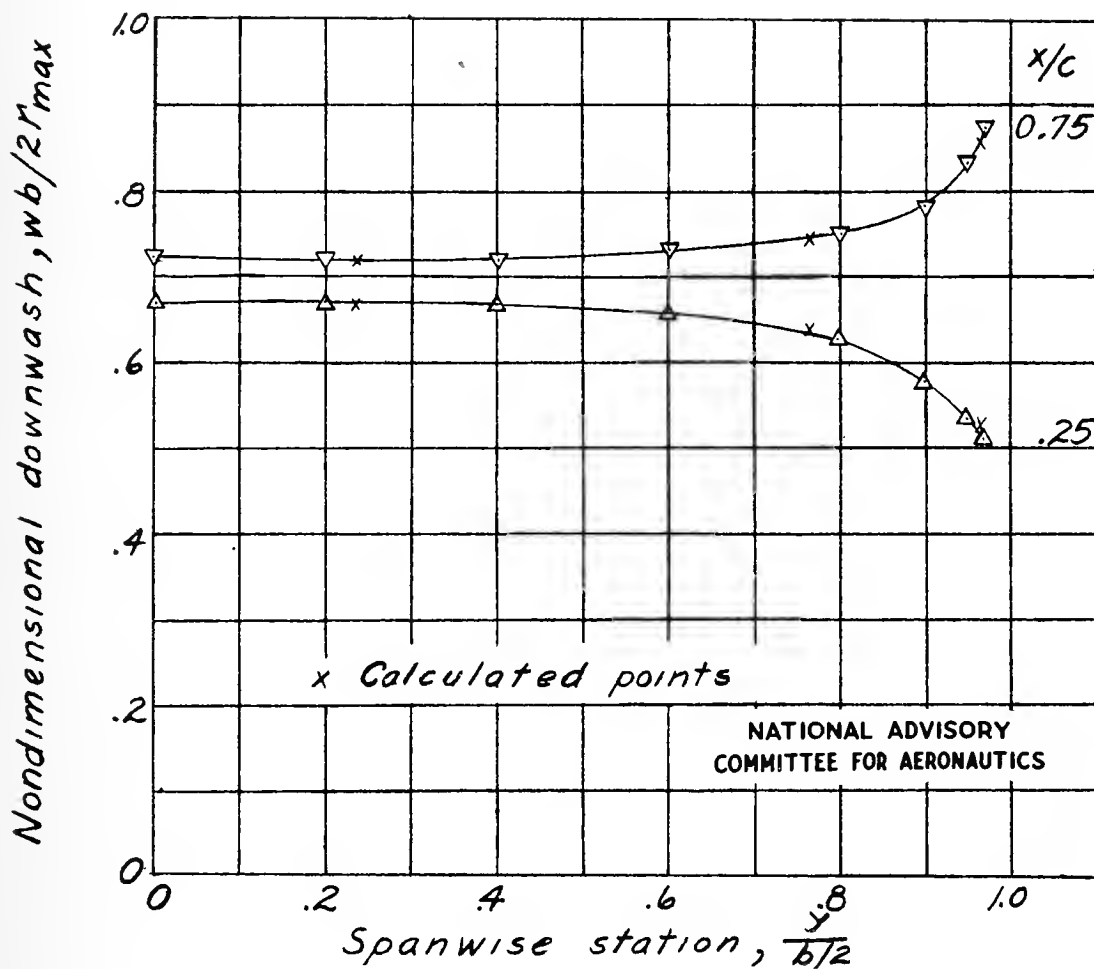
Figure 12.—The nondimensional downwash $\dfrac{wb}{2\Gamma_{max}}$ for an elliptic wing having an aspect ratio of 3 with flat-plate type of loading. Comparison of results obtained by method of reference 2 with data, extrapolated to $z = 0$, obtained from tests of preliminary electromagnetic-analogy model.